Efficiency of the lever
(for DPC application)

Actually Discrete Power Converter (DPC or Kornich machine) operates as lever powered by gravity during its single stroke. Thus, analysis of lever efficiency can be useful for understanding of estimation of efficiency of DPC.

This substitution is presented by illustration on Fig.1

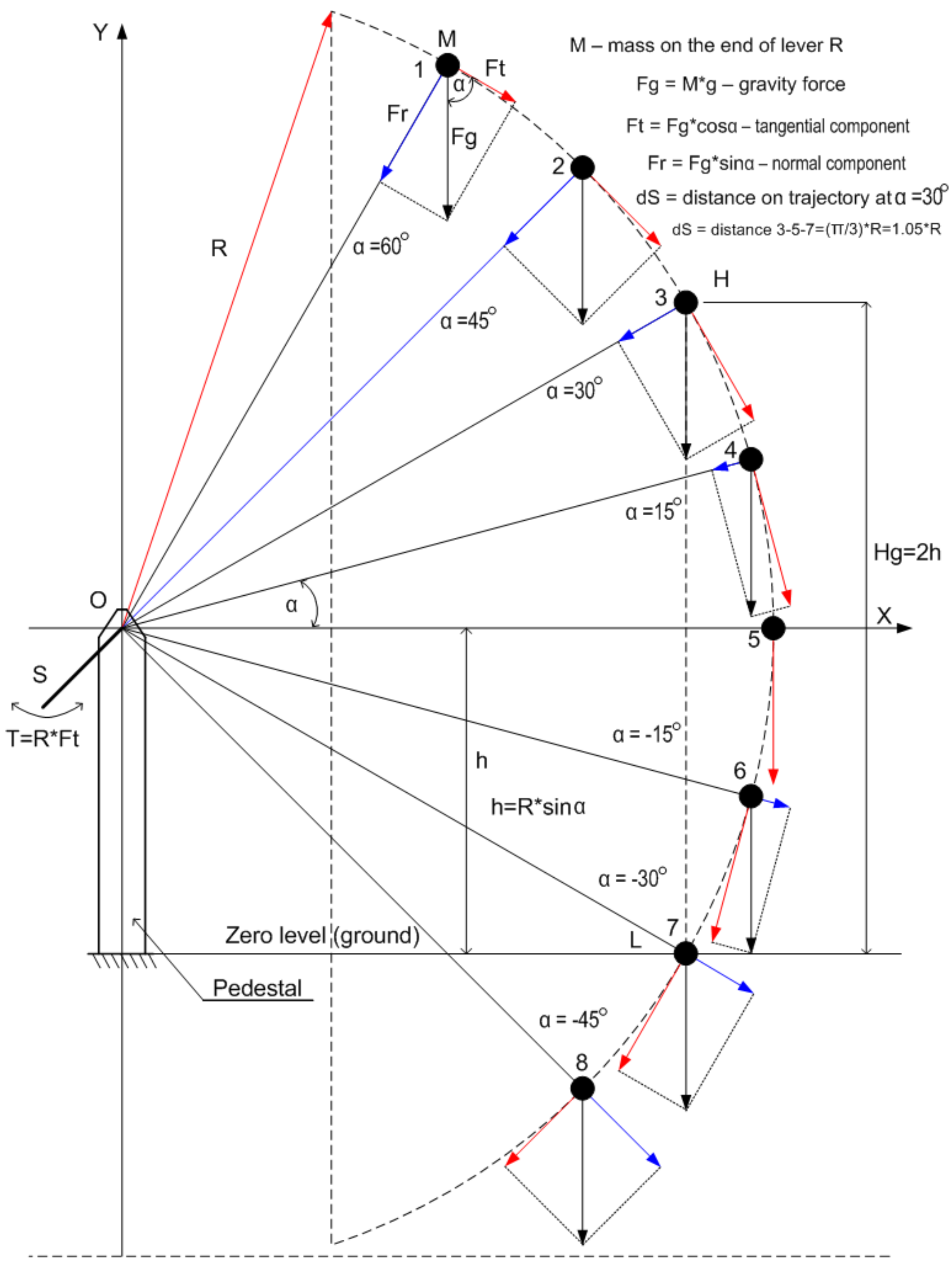

Fig.1

In that set up the rigid rod (**R** - as lever itself) is attached tightly to horizontal shaft **S** and this assembly fixed at pivot point **O** on the pedestal.

To simplify the analysis we are applying the next admissions:

1. mass M at the end of lever determines the whole its mass (mass of rod is negligible),
2. rod is rigid absolutely (no deformation exist),
3. friction at pivot point **O** is absent (no losses of energy),
4. the angle **α** for lever **R** above and below of horizontal axis **X** is equal and symmetrical and presents the main variable value in our considering,
5. zero level (ground) is function from angle **α** to provide the condition 4 in the lowest position **L** of lever

When the lever will be raised to high position **H** at some angle (**α** = 30* on Fig.1, for example) and then will be released, the end of lever (point **M**) will travel on arc distance **dS** from point **H** to point **L** (very similar to operational stroke of DPC) forced by gravity:

$$Fg = M*g \qquad (1)$$

Fg – value of force of gravity
M – mass on th end of lever
g – gravitational constant

On each point of trajectory (between points **3** and **7**) at each moment of time of travelling the gravity force can be presented by two orthogonal components:

the tangential component

$$Ft = Fg * \cos\boldsymbol{\alpha} \qquad (2)$$

which moves mass **M** along arc trajectory;

the normal component

$$Fr = Fg * \sin\boldsymbol{\alpha} \qquad (3)$$

which acts along rod of lever and caused its deformation

By moving mass **M** from high position **H** (point **3**, for example) to low position **L** (point 7, for example) tangential component performs the work

$$A = Ft * dS \qquad (4)$$

A – value of work
dS – distance of travelling of mass M along arc trajectory (between point **3** and **7**, for example)

Length of arc at central angle 2**α** can be determined as part of full revolution of rod R (see Fig.1)

dS = 2**α**(rad) *R = [2**α**(rad)] * R (5)

or **dS** = η * R (6)

where η = 2**α**(rad) - dimensionless coefficient proportional to angle **α** (7)

From (4) and (1); (2);(5);(6)

$$A = Fg * \cos\boldsymbol{\alpha} * \eta * R = M * g * \cos\boldsymbol{\alpha} * \eta * R \qquad (8)$$

Before beginning the motion from point **H** (point 3, for example) the mass **M** posses potential energy at gravitational "head" **Hg** (as difference between points **3** and **7**)

From Fig.1 evidently

$$Hg = 2 * h = 2 * R * \sin\boldsymbol{\alpha} \qquad (9)$$

Therefore, the potential energy Ep accumulated by mass M at “head” **Hg** is determined as:

$$Ep = Fg * Hg = M * g * Hg \qquad (10)$$

and substituting (9)

$$Ep = 2 * M * g * R * \sin\boldsymbol{\alpha} \qquad (11)$$

Actually, the mechanical work **A** is performed by transforming of potential energy **Ep** into kinetic energy of moving mass **M.** Hence in the terms of operating of lever as simple machine we can consider Ep as input value and A as output value. Thus, the efficiency of lever can be determined as:

$$\boldsymbol{\varepsilon} = A / Ep = = [M * g * \cos\boldsymbol{\alpha} * \eta * R] / [2 * M * g * R * \sin\boldsymbol{\alpha}] = (\eta * ctg\boldsymbol{\alpha}) / 2 = [2\boldsymbol{\alpha}(rad) * ctg\boldsymbol{\alpha}]/2 \qquad (12)$$

Finally,

$$\boldsymbol{\varepsilon} = \boldsymbol{\alpha}(rad) * ctg\boldsymbol{\alpha} \qquad (13)$$

The results of calculation by formula (13) for variable angle of initial position of lever (accordingly to Fig.1) are presented in Table 1 and chart Fig.2.

Table 1

angle (grad)	60	55	50	45	40	35	30	25	20	15	10	5
angle (rad)	1.05	0.96	0.87	0.79	0.70	0.61	0.52	0.44	0.35	0.26	0.17	0.09
effic	0.604	0.672	0.732	0.785	0.830	0.873	0.905	0.933	0.959	0.976	0.989	0.997

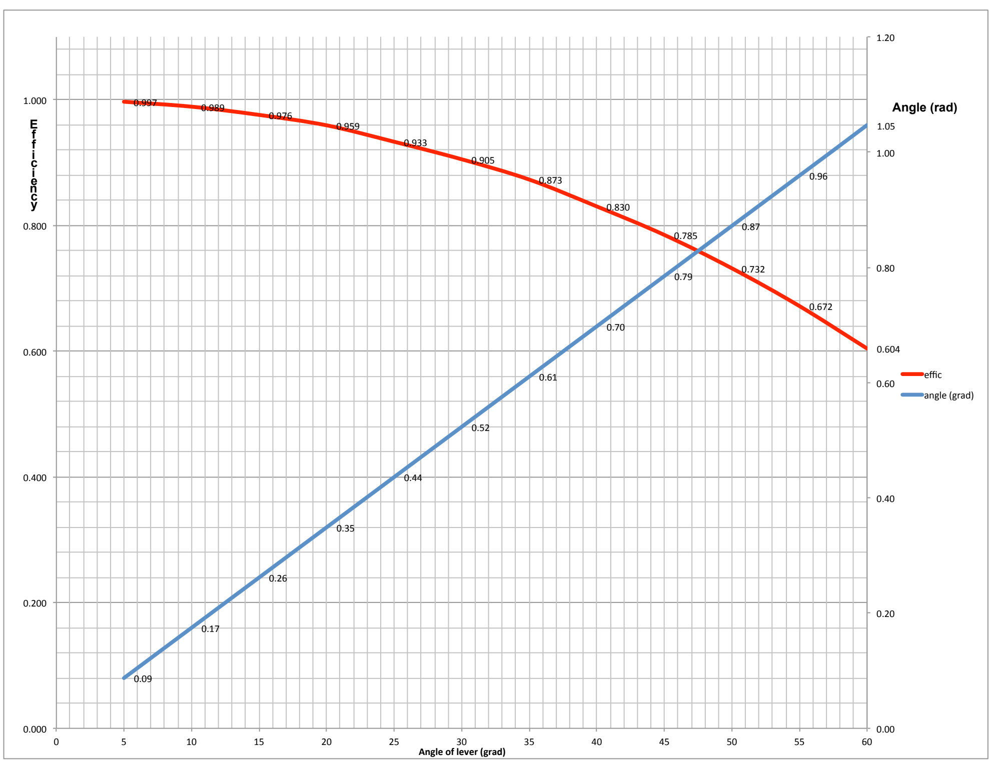

Fig.2

Conclusion:

1. The analysis of motion of lever around pivot point **O** under gravity force (and with admissions 1-5) can be used for estimation of theoretical values of efficiency of DPC (*Kornich machine*) depending on initial position (angle **α**) of rocking beam-lever in the construction of machine.
2. Only tangential component (**Ft)** of gravity force can be transformed into kinetic energy of mass M (on the end of lever) during travelling lever along arc trajectory from high position (H) to low position (L)
3. The kinetic energy of lever allows to perform some useful mechanical work under appropriate mechanical loads with corresponded kind of required motion (for example, piston pump).
4. The normal component (**Fr**) of gravity force can not perform of mechanical work, instead it try to deform the lever itself. During one stroke the normal component changes its direction (above the horizontal level component **Fr** compresses the lever **R** and below – tries to stretch it). This phenomena should be taken in account in the choosing of right material for beam-lever in construction of machine.
5. The efficiency of falling lever (calculated by 13 equation) has trend to be increased with decreasing of angle **α**, this is because of tangential component (**Ft)** is increased by equation (2). Thus the maximum efficiency is presented in point (5) of trajectory, when angle **α** is equal zero and **Ft = Fg**
6. The range for angle **α** = (30 – 20) grad is optimal for construction of machine (at angle **α** less than 20 degree the space below lever is limited for placing of practical mechanical loads).